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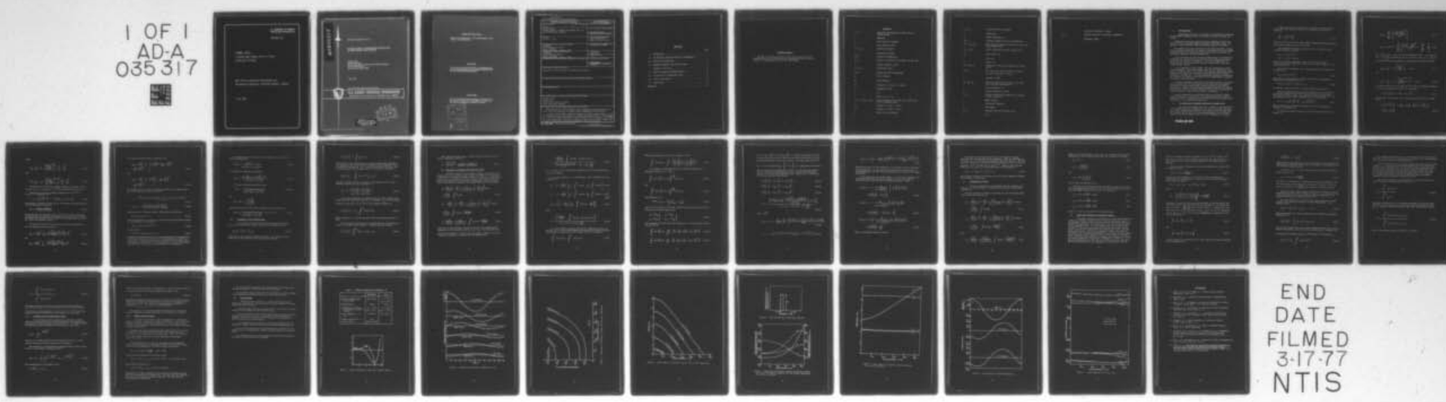

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THERMAL STRESS

A NARROW BAND RANDOM LOAD IN A SOLID PROPELLANT CYLINDER

ARMY MISSILE RESEARCH, DEVELOPMENT AND ENGINEERING LABORATORY, REDSTONE ARSENAL, ALABAMA

7 MAY 1976

TECHNICAL REPORT RK-7T-1

THERMAL STRESS: A NARROW BAND RANDOM LOAD IN A SOLID PROPELLANT CYLINDER

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7 May 1976

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The stresses in a large diameter solid propellant cylinder encased in a steel shell and subjected to random surface temperatures are analyzed.

A plane strain elastic solution coupled with random process theory is used to determine the mean and variance of thermal stresses as well as the response to a sinusoidal input temperature in the interior of the cylinder. It is shown that stress components are nearly constant in amplitude at various depths and that the diurnal cycle produces insignificant stress variations.

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CONTENTS

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The Kelvin functions utilized in these calculations have been programmed by Mr. John Sofferis of the Redstone Army Arsenal Computer Center. His cooperation is gratefully acknowledged.

SYMBOLS

a, $\bar{a}$	amplitude; mean amplitude; inner radius of hollow cylinder
A	amplitude
b	outer radius of cylinder
c	outer radius of case
C, C_1, C_2	arbitrary constants
e	exponential function
$E[U^2]$	variance of temperature
E, E_c	modulus of elasticity of cylinder, of case, psi
$\bar{f}$	average frequency, cy/hr
f, f_1, f_2	frequencies, cy/hr
$f_A(a)$	density function of amplitudes
f_m	Kelvin function
g_m	Kelvin function
$h(.)$	function $h(\cdot) = \mathrm{ber}(\cdot) + i\,\mathrm{bei}(\cdot)$
h_c	thickness of case
i	$\sqrt{-1}$
j	index ($j = r, \theta, z$)
$J_O(\cdot)$, $J(\cdot)_{OR}$, $J(\cdot)_{OI}$	Bessel function of the first king, order zero; real and imaginary parts
$k(\cdot)$	function, $k = \mathrm{ber}(\cdot) - \mathrm{bei}(\cdot)$
$\ell(\cdot)$	function, $\ell = \mathrm{ber}(\cdot) + \mathrm{bei}(\cdot)$
m	order of Kelvin function

p', p'_m	function in stress equations
P	Probability
r	radial coordinate, in.
$R(\cdot)$	frequency response function of temperature
$\bar{S}(.)$, $\bar{S}_{u_i}$, $\bar{S}_{u_O}$	power spectral density functions; for input and output temperatures
S_r, S_θ, S_z	radial transverse and axial stresses, psi
S_m	mean stress, psi
t	time, hrs
T	period, hrs
$U(r)$, U_m, U_y	temperature; annual mean; amplitude of annual cycle
W_i, W_O	one sided power spectral density function; for input and output, $°F^2$/cy/hr
x	variable $x = \sqrt{\frac{\omega}{\alpha}}\, r$
Y_O, Y_{OR}, Y_{OI}	Bessel function of the second kind of order zero; real and imaginary parts
z	axial coordinate, in.
α	thermal diffusivity, $in.^2$/hr
$\bar{\alpha}$, $\bar{\alpha}_c$	thermal coefficient of expansion in cylinder; in case, in./in./°F
Γ	gamma function
∂	differential operator
ϕ	phase angle
ν, ν_c	Poisson's ratio for cylinder; case
π	3.14

θ	transverse coordinate, radians
σ_a, σ_u	standard deviation of amplitude; temperature
ω	frequency, rad/hr

I. INTRODUCTION

Engineering structures are subject to environmental variations of the ambient temperature that in turn produce thermal stresses and strains.

Temperature variations consist of cyclic components (annual and diurnal cycles) and random components produced by clouds, winds, rain, and snow. Solar radiation creates additional temperature changes.

The response of a large structure or structural component, having various thermal properties, will depend on the frequency of such changes. The temperature at the interior of the structure will be delayed and attenuated compared to surface temperatures.

A slow variation such as the annual summer-winter cycle penetrates throughout the structure while the diurnal cycle effects only surface layers or thin structural components. As a result temperatures in structures are usually nonuniform and produce nonuniform cyclic and random thermal stresses.

In the following a large diameter circular cylinder with casing will be examined. The annual thermal cycle will be considered to be a slowly varying mean while the diurnal cycle and the superimposed random variations are assumed to be narrow band excitations with a central frequency of one per day. The large difference between the frequencies of the annual and diurnal cycles makes it not unreasonable to consider the process to be stationary.

The mean and standard deviation of the thermal stress at any point in the cylinder will be calculated from the given mean and standard deviation of surface temperatures which are assumed to be Gaussian distributed. Because the governing differential equations for temperature conduction and for thermal stress are linear, the distribution of thermal stress at a point is also going to be Gaussian.

Once the parameters of the distribution are calculated a safety factor analysis may be performed if the distribution of the material strength is known. Because loads are repeated the safety analysis will also lead to a life prediction capability.

II. THE FREQUENCY RESPONSE FUNCTION OF TEMPERATURE

The frequency response function is defined as the response to a sinusoidal input of unit amplitude. For the case at hand the input is a cyclic temperature variation at the surface of the cylinder while the response may be either the resulting temperature at an interior point or any of the thermal stress or strain components at the point.

A long axisymmetric cylinder will be analyzed for which the one dimensional heat conduction equation in cylindrical coordinates is valid [1]

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = \frac{1}{\alpha}\frac{\partial U}{\partial t} \qquad \text{(II-1)}$$

where $U(t,r)$ is the time and space (radial) dependent temperature and α is the thermal diffusivity in in.^{2}/hr (m^2/S).

The differential equation, Equation (II-1), is subject to the following boundary conditions

$$\text{BC.1} : U(b,t) = e^{i\omega t}$$

$$\text{BC.2} : U(0,t) = \text{finite} \quad .$$

That is a sinusoidal temperature input with circular frequency, ω radians/hr, and unit amplitude is applied at the outer radius, b, of a solid cylinder [2].

The solution to Equation (II-1) using separation of variables may be written as

$$U(r,t) = R(r,\omega)\, e^{i\omega t} \qquad \text{(II-2)}$$

where $R(r,\omega)$ is the frequency response function of the temperature. Substituting into Equation (II-1)

$$R''(r,\omega) + \frac{1}{r} R'(r) - \frac{i\omega}{\alpha} R(r,\omega) = 0 \qquad \text{(II-3)}$$

is obtained. Equation (II-3) is a Bessel differential equation.

The above problem has been solved by Dahl as far back as 1924 for a caseless cylinder with the use of Bessel and Neuman functions[3]. Here Kelvin functions, which are essentially Bessel functions with complex arguments, will be used. Equation (II-3) is satisfied by

$$R(r,\omega) = C_1 J_0\left(\sqrt{-\frac{i\omega}{\alpha}}\, r\right) + C_2 Y_0\left(\sqrt{-\frac{i\omega}{\alpha}}\, r\right) \quad . \qquad \text{(II-4)}$$

These functions are described by Wattson [4] and by the Handbook of Mathematical Functions [5].

In series form Bessel functions of the first and second kind are written as

$$J_0(x) = \sum_{n=1}^{\infty} \left[\frac{(-1)^n \left(\frac{1}{2}x\right)^{2n}}{(n!)^2} \right] , \qquad \text{(II-5)}$$

$$Y_0(x) = J_0(x) \ln x - \sum_{n=1}^{\infty} \left[\frac{(-1)^n \left(\frac{1}{2}x\right)^{2n}}{(n!)^2} \sum_{n=1}^{n} \frac{1}{j} \right], \qquad \text{(II-6)}$$

and, in this case, consist of real and imaginary series. Substituting $\sqrt{-i} = i\sqrt{i} = \frac{i-1}{\sqrt{2}}$ into Equations (II-5) and (II-6) and using $x = \sqrt{\frac{\omega}{\alpha}}\, r$ as the variable the real and imaginary parts become the Kelvin functions

$$J_{OR}(x) = \sum_{n=1}^{\infty} (-1)^n \left[\frac{\left(\frac{x}{2}\right)^{2n}}{(2n)!} \right]^2 = \text{ber } x \qquad \text{(II-7)}$$

and

$$J_{OI}(x) = \sum_{n=1}^{\infty} (-1)^n \left[\frac{\left(\frac{x}{2}\right)^{(2n+1)}}{(2n+1)!} \right]^2 = \text{bei } x \quad . \qquad \text{(II-8)}$$

The Bessel function of the second kind may also be developed analogously. It should be recognized that $\frac{i\pi}{2} = \ln i$ and hence

$$\ln \left(i\sqrt{i}\sqrt{\frac{\omega}{\alpha}}\, r \right) = \frac{3i\pi}{4} + \ln \sqrt{\frac{\omega}{\alpha}}\, r \quad . \qquad \text{(II-9)}$$

Substituting into Equation (II-6) and using Equations (II-7) through (II-9)

$$Y_0 \left(i\sqrt{i}\sqrt{\frac{\omega}{\alpha}}\, r \right) = \left(J_{OR} + iJ_{OI} \right) \left(-\frac{\pi i}{4} + \ln \sqrt{\frac{\omega}{\alpha}}\, r \right)$$

$$- \left(\bar{Y}_{OR} + i\bar{Y}_{OI} \right) \qquad \text{(II-10)}$$

where

$$\overline{Y}_{OR} = \sum_{n=0}^{\infty} (-1)^n \left\{ \left[\frac{\left(\frac{x}{2}\right)^{2n}}{(2n)!} \right]^2 \sum_{j=1}^{2n} \frac{1}{j} \right\} \tag{II-11}$$

and

$$\overline{Y}_{OI} = \sum_{n=0}^{\infty} (-1)^n \left\{ \left[\frac{\left(\frac{x}{2}\right)^{(2n+1)}}{(2n+1)!} \right]^2 \sum_{j=1}^{(2n+1)} \frac{1}{j} \right\} \quad . \tag{II-12}$$

Introduction of the second boundary condition in Equation (II-10) requires the $C_2 = 0$ because $Y_0 \left(i\sqrt{i}\sqrt{\frac{\omega}{\alpha}}\, r\right)$ approaches infinity at $r = 0$.

Making use of the first boundary condition $U(b,t) = e^{i\omega t}$ or $R(b,\omega) = 1$ in Equation (II-4).

$$1 = C_1 \, J_0 \left(\sqrt{-\frac{i\omega}{\alpha}}\; b\right) = C_1 \left(\text{ber } x_b + i \text{ bei } x_b\right) \quad . \tag{II-13}$$

The solution of Equation (II-4) that is the frequency response function of the temperature is therefore

$$R(r,\omega) = \frac{\text{ber } x + i \text{ bei } x}{\text{ber } x_b + i \text{ bei } x_b} \quad . \tag{II-14}$$

Kelvin functions of order zero, ber x and bei x, as well as functions of higher order are tabulated in Reference [5] and are shown in Figure 1. The tables also contain Kelvin functions of the second kind, ker x and kei x which are related to $Y_0(x)$.

The higher order Kelvin functions $\text{ber}_m\, x$ and $\text{bei}_m\, x$ needed here are also expressed in series form [5]

$$\text{ber}_m\, x = \left(\frac{1}{2}x\right)^m \sum_{k=0}^{\infty} \frac{\cos\left[\left(\frac{3}{4}m + \frac{1}{2}k\right)\pi\right]}{k!\;\Gamma\,(m+k+1)} \left(\frac{1}{4}\,x^2\right)^k \tag{II-15}$$

and

$$\text{bei}_m\, x = \left(\frac{1}{2}x\right)^m \sum_{k=0}^{\infty} \frac{\sin\left[\left(\frac{3}{4}m + \frac{1}{2}k\right)\pi\right]}{k!\;\Gamma\,(m+k+1)} \left(\frac{1}{4}\,x^2\right)^k \quad . \tag{II-16}$$

The first few terms of $\mathrm{ber}_1\, x$ and $\mathrm{bei}_1\, x$ are

$$\mathrm{ber}_1\, x = \frac{\sqrt{2}}{4}x \left[- 1 - \frac{1}{2}\left(\frac{x^2}{4}\right) + \frac{1}{2!3!}\left(\frac{x^2}{4}\right)^2 + \frac{1}{3!4!}\left(\frac{x^2}{4}\right)^3 \ldots\right] \tag{II-17}$$

and

$$\mathrm{bei}_1\, x = \frac{\sqrt{2}}{4}x \left[1 - \frac{1}{2}\left(\frac{x^2}{4}\right) - \frac{1}{2!3!}\left(\frac{x^2}{4}\right)^2 + \frac{1}{3!4!}\left(\frac{x^2}{4}\right)^3 + \ldots\right] . \tag{II-18}$$

The magnitude of the frequency response function and its phase angle may be obtained from Equation (II-14) as

$$|R(r,\omega)| = \frac{\left[\left(\mathrm{ber}\, x\ \mathrm{ber}\, x_b + \mathrm{bei}\, x\ \mathrm{bei}\, x_b\right)^2 + \left(\mathrm{bei}\, x\ \mathrm{ber}\, x_b - \mathrm{ber}\, x\ \mathrm{bei}\, x_b\right)^2\right]^{1/2}}{\mathrm{ber}^2\, x_b + \mathrm{bei}^2\, x_b} \tag{II-19}$$

$$\tan \theta(r,\omega) = \frac{\mathrm{bei}\, x\ \mathrm{ber}\, x_b - \mathrm{ber}\, x\ \mathrm{bei}\, x_b}{\mathrm{ber}\, x\ \mathrm{ber}\, x_b + \mathrm{bei}\, x\ \mathrm{bei}\, x_b} . \tag{II-20}$$

Consequently the frequency response function may be written as

$$R(r,\omega) = |R(r,\omega)|\ e^{i\theta} \tag{II-21}$$

and the temperature at a point as a function of time and radial position is from Equation (II-2) given by

$$U(r,t) = |R(r,\omega)|\ e^{i(\theta+\omega t)} . \tag{II-22}$$

The real part

$$U(r,t) = |R(r,\omega)|\ \cos\,(\theta + \omega t) \tag{II-23}$$

indicates the time delay as well as the attenuation of the temperature as shown in Figure 2 for a 22 in. (55.9 cm) propellant cylinder with steel casing. Figures 3 and 4 show the amplitude and the phase angle (time delay) in the interior of such cylinders; it should be remembered that both $|R(r,\omega)|$ and $\theta(r,\omega)$ are functions of the frequency, ω.

The temperature at the center of the cylinder, at r=0 (ber 0=1 and bei 0=0) becomes

$$U(0,t) = \frac{R^{i\omega t}}{\text{ber}\sqrt{\frac{\omega}{\alpha}}b + i\ \text{bei}\sqrt{\frac{\omega}{b}}b} \tag{II-24}$$

or removing the imaginary denominator

$$U(0,t) = \frac{(\text{ber}\sqrt{\frac{\omega}{\alpha}}b - i\ \text{bei}\sqrt{\frac{\omega}{\alpha}}b)\ e^{i\omega t}}{\text{ber}^2\sqrt{\frac{\omega}{\alpha}} + \text{bei}^2\sqrt{\frac{\omega}{\alpha}}b} \ . \tag{II-25}$$

In terms of magnitude and phase angle

$$U(0,t) = \frac{1}{\sqrt{\text{ber}^2\sqrt{\frac{\omega}{\alpha}}b + \text{bei}^2\sqrt{\frac{\omega}{\alpha}}b}} \tag{II-26}$$

and

$$\tan\theta(0,t) = -\frac{\text{bei}\sqrt{\frac{\omega}{\alpha}}b}{\text{ber}\sqrt{\frac{\omega}{\alpha}}b}$$

hence the real part of the central temperature may be written as

$$U(0,t)_R = \frac{1}{\sqrt{\text{ber}^2\sqrt{\frac{\omega}{\alpha}}b + \text{bei}^2\sqrt{\frac{\omega}{\alpha}}b}}\ \cos(\omega t + 0)\ . \tag{II-27}$$

III. VARIANCE OF THE TEMPERATURE

For a random excitation with input power spectral density, $S_{u_i}(\omega)$, the output power spectral density may be obtained as [6]

$$\overline{S}_{u_0}(\omega) = |R(\omega)|^2\ \overline{S}_{u_i}(\omega) \tag{III-1}$$

where $R(\omega)$ is the frequency response function. The variance of the output can be calculated from the power spectral density

$$E\left[U_0^{\,2}(r)\right] = \int_{-\infty}^{\infty} \overline{S}_{u_0}(\omega)\, d\omega \quad . \tag{III-2}$$

The variance of the temperature at a point is consequently obtained from Equations (III-1) and (III-2) by substitution of Equation (II-19) and numerical integration. For a narrow band random excitation, such as that produced by the diurnal cycle ($\omega_0 = 2\pi f_0$, $f_0 = 1/24$ hrs),

$$E\left[U_0^{\,2}(r)\right] = \int_{-\infty}^{\infty} |R(r,\omega)|^2\, \overline{S}_{u_i}(\omega)\, d\omega \quad . \tag{III-3}$$

Expanding Equation (II-19) the square of the absolute value of the frequency response function simplifies to

$$|R(r,\omega)|^2 = \frac{\mathrm{ber}^2 \sqrt{\frac{\omega}{\alpha}}\,r + \mathrm{bei}^2 \sqrt{\frac{\omega}{\alpha}}\,r}{\mathrm{ber}^2 \sqrt{\frac{\omega}{\alpha}}\,b + \mathrm{bei}^2 \sqrt{\frac{\omega}{\alpha}}\,b} \quad . \tag{III-4}$$

If the input temperature is considered to be a band limited white noise whose one sided power spectrum is a constant, W_{u_i}, over the narrow frequency range from f_1 to f_2 and is zero elsewhere (Figure 5)

$$E\left[U_0^{\,2}(r)\right] = W_{u_i} \int_{f_1}^{f_2} |R(r,f)|^2\, df \tag{III-5}$$

where frequency, f, in cycles per hour rather than radians per hour is used.

At the surface the variance of the temperature may be expressed in terms of the power spectral density. Based on Equation (III-2),

$$E\left[U_i^{\,2}(0)\right] = \int_{f_1}^{f_2} W_i\, df = W_i\,(f_2 - f_1) \quad . \tag{III-6}$$

Hence the input power spectral density may be calculated from the surface temperature variance as

$$W_i = \frac{E\left[U_i^2(0)\right]}{f_2 - f_1} \quad \frac{\deg F^2}{\text{cycles/hr}} \left(\frac{\deg C^2}{\text{cycles/sec}}\right) . \tag{III-7}$$

IV. FREQUENCY RESPONSE FUNCTION FOR STRESS

Thermal stresses in long cylinders (plane strain) have been developed by several authors such as Boley and Wiener [7] and have been calculated for a caseless cylinder subjected to sinusoidal temperatures by Dahl [3]. A cylinder with a thin case has been treated by Williams, Blatz, and Schapery [8] for arbitrary temperatures. Their relations will be used here for sinusoidal and random temperature distributions.

$$S_r = \frac{b^2 p'}{b^2 - a^2}\left(1 - \frac{a^2}{r^2}\right) + \frac{\bar{\alpha} E}{(1-\nu)(b^2-a^2)}\left(1 - \frac{a^2}{r^2}\right)\int_a^b U(r)r\,dr - \frac{\bar{\alpha} E}{(1-\nu)r^2}\int_a^r U(r)r\,dr \tag{IV-1}$$

$$S_\theta = -\frac{b^2 p'}{b^2-a^2}\left(1 + \frac{a^2}{r^2}\right) + \frac{\bar{\alpha} E}{(1-\nu)(b^2-a^2)}\left(1 + \frac{a^2}{r^2}\right)\int_a^b U(r)r\,dr + \frac{\bar{\alpha} E}{(1-\nu)r^2}\int_a^r U(r)r\,dr - \frac{\bar{\alpha} E\, U(r)}{(1-\nu)} \tag{IV-2}$$

$$S_z = \frac{2\nu b^2 p'}{(b^2-a^2)} + \frac{2\nu\bar{\alpha} E}{(1-\nu)(b^2-a^2)}\int_a^b U(r)r\,dr - \frac{\alpha\nu E U(r)}{(1-\nu)} \tag{IV-3}$$

where U(r) is the difference between the stress free (construction) temperature and its momentary value, a and b are the inner and outer radii of the cylinder E, ν and $\bar{\alpha}$ are the modulus, Poisson's ratio and a thermal coefficient of expansion for the material and

$$p' = \frac{E\,\dfrac{2\bar{\alpha}(1+\nu)}{(b^2-a^2)} \displaystyle\int_a^b U(r)rdr - \bar{\alpha}_c U(b)\,(1+\nu_c)\,E}{\dfrac{(1+\nu)\left[(1-2\nu)\,b^2 + a^2\right]}{b^2 - a^2} + \left(1 - \nu_c^{\,2}\right)\dfrac{bE}{h_c E_c}} \quad . \tag{IV-4}$$

E_c, ν_c, and $\bar{\alpha}_c$ are the mechanical parameters of the case materials and h_c is its thickness.

For a solid cylinder a = 0 and Equations (IV-1) through (IV-4) are simplified to

$$S_r = -\,p' + \frac{\bar{\alpha}E}{(1-\nu)}\left[\frac{1}{b^2}\int_0^b rU(r)dr - \frac{1}{r^2}\int_0^r rU(r)dr\right] \tag{IV-5}$$

$$S_\theta = -\,p' + \frac{\bar{\alpha}E}{(1-\nu)}\left[\frac{1}{b^2}\int_0^b rU(r)dr + \frac{1}{r^2}\int_0^r rU(r)dr - U(r)\right] \tag{IV-6}$$

$$S_z = 2\nu\left\{p' + \frac{\bar{\alpha}E}{(1-\nu)}\left[\frac{1}{b^2}\int_0^b rU(r)rdr - \frac{U(r)}{2}\right]\right\} \tag{IV-7}$$

with

$$p' = \frac{E\left[\dfrac{2\bar{\alpha}(1+\nu)}{b^2}\displaystyle\int_0^b U(r)rdr - \bar{\alpha}_c\,(1+\nu_c)\,U(b)\right]}{(1+\nu)\,(1-2\nu) + (1-\nu_c^{\,2})\,\dfrac{bE}{h_c E_c}} \quad . \tag{IV-8}$$

Given the frequency response function for temperature, $R(r,\omega)$ of Equation (II-14), the stress response functions may be calculated after the integrals in Equations (IV-5) through (IV-8) are evaluated

$$\int_0^r rU(r,\omega)dr = \int_0^r r\;\; R(r,\omega)\;dr \quad . \tag{IV-9}$$

Substituting Equation (II-14) into Equation (IV-9)

$$\int_0^r rU(r,\omega)dr = \int_0^r r \frac{\left[\text{ber}\sqrt{\frac{\omega}{\alpha}}\, r + i\ \text{bei}\sqrt{\frac{\omega}{\alpha}}\, r\right]}{\text{ber}\sqrt{\frac{\omega}{\alpha}}\, b + i\ \text{bei}\sqrt{\frac{\omega}{\alpha}}\, b}\, dr \quad . \tag{IV-10}$$

The integrals of Equation (IV-10) are evaluated individually. Changing variables to $x = \sqrt{\frac{\omega}{\alpha}}\, r$

$$\int_0^r r\ \text{ber}\sqrt{\frac{\omega}{\alpha}}\ rdr = \frac{\omega}{\alpha} \int_0^{\sqrt{\frac{\omega}{\alpha}}r} x\ \text{ber}\ x\ dx \tag{IV-11}$$

and

$$\int_0^r r\ \text{bei}\sqrt{\frac{\omega}{\alpha}}\ rdr = \frac{\omega}{\alpha} \int_0^{\sqrt{\frac{\omega}{\alpha}}r} x\ \text{bei}\ x\ dx \quad . \tag{IV-12}$$

From Reference [5]

$$x^{(1+m)}\ f_m\ dx = -\frac{x^{1+m}}{\sqrt{2}}\left(f_{m+1} - g_{m+1}\right) \tag{IV-13}$$

where m is the order of the Kelvin function and f_m and g_m are any one of the pairs of functions given below

$$\left.\begin{array}{l} f_m = \text{ber}_m x \\ g_m = \text{bei}_m x \end{array}\right\} \quad \left.\begin{array}{l} f_m = \text{bei}_m x \\ g_m = -\ \text{ber}_m x \end{array}\right\} \quad . \tag{IV-14}$$

Hence Equations (IV-11) and (IV-12) become (m = 0 is not usually written: ber $_0x$ = ber x)

$$\int_0^r r\ \text{ber}\sqrt{\frac{\omega}{\alpha}}\ rdr = \frac{-\alpha}{\omega\sqrt{2}}\ \sqrt{\frac{\omega}{\alpha}}r\ \left(\text{bei}_1\sqrt{\frac{\omega}{\alpha}}r + \text{ber}_1\sqrt{\frac{\omega}{\alpha}}\ r\right) \tag{IV-15}$$

$$\int_0^r r\ \text{bei}\sqrt{\frac{\omega}{\alpha}}\ rdr = \frac{-\alpha}{\omega\sqrt{2}}\ \sqrt{\frac{\omega}{\alpha}}r\ \left(\text{bei}_1\sqrt{\frac{\omega}{\alpha}}\ r + \text{ber}_1\sqrt{\frac{\omega}{\alpha}}\ r\right) . \tag{IV-16}$$

At $r = 0$ $\mathrm{ber}_1 \sqrt{\frac{\omega}{\alpha}}\, r = 0$ and $\mathrm{bei}_1 \sqrt{\frac{\omega}{\alpha}}\, r = 0$ hence both integrals vanish at the lower limit as can be seen from the series definitions of $\mathrm{ber}_1 x$ and $\mathrm{bei}_1 x$ [Equations (II-15) and (II-16)]. The same series show that at $r = 0$, the expression $\frac{1}{r^2} \int_0^r r\, \mathrm{ber} \sqrt{\frac{\omega}{\alpha}}\, r\, dr$, appearing in Equations (IV-5) through (IV-8) approaches the value of $+1/2$ while the analogus term for the bei function approaches zero. Substituting Equations (IV-15) and (IV-16) into Equations (IV-5), (IV-6), (IV-7), (IV-8) and (IV-10) and using the abbreviations

$$h\left(\sqrt{\frac{\omega}{\alpha}}\, r\right) = \mathrm{ber} \sqrt{\frac{\omega}{\alpha}}\, r + i\, \mathrm{bei} \sqrt{\frac{\omega}{\alpha}}\, r \tag{IV-17}$$

$$k\left(\sqrt{\frac{\omega}{\alpha}}\, r\right) = \mathrm{ber}_1 \sqrt{\frac{\omega}{\alpha}}\, r - \mathrm{bei}_1 \sqrt{\frac{\omega}{\alpha}}\, r \tag{IV-18}$$

$$\ell\left(\sqrt{\frac{\omega}{\alpha}}\, r\right) = \mathrm{ber}_1 \sqrt{\frac{\omega}{\alpha}}\, r + \mathrm{bei}_1 \sqrt{\frac{\omega}{\alpha}}\, r \tag{IV-19}$$

$$p'(\omega) = -\frac{\left\{E\, \frac{2\bar{\alpha}\,(1+\nu)}{b} \sqrt{\frac{\alpha}{2\omega}}\, \frac{\left[k\left(\sqrt{\frac{\omega}{\alpha}}\, b\right) + i\ell\left(\sqrt{\frac{\omega}{\alpha}}\, b\right)\right]}{h\left(\sqrt{\frac{\omega}{\alpha}}\, b\right)} + \bar{\alpha}_c\,(1+\nu_c)\right\}}{(1+\nu)\,(1-2\nu) + (1-\nu_c^{\,2})\, \frac{bE}{h_c E_c}} \tag{IV-20}$$

and $x = \sqrt{\frac{\omega}{\alpha}}\, r$

$$S_r(r,\omega) = -p' + \frac{\bar{\alpha}E}{(1-\nu)} \sqrt{\frac{\alpha}{2\omega}} \left\{\frac{-\frac{1}{b}\left[k(x_b) + i\ell(x_b)\right] + \frac{1}{r}\left[k(x) + i\ell(x)\right]}{h(x_b)}\right\} \tag{IV-21}$$

$$S(r,\omega) = -p' - \frac{\bar{\alpha}E}{(1-\nu)} \sqrt{\frac{\alpha}{2\omega}} \left\{\frac{\frac{1}{b}\left[k(x_b) + i\ell(x_b)\right] + \frac{1}{r}\left[k(x) + i\ell(x)\right] + \sqrt{\frac{2\omega}{\alpha}}\, h(x)}{h(x_b)}\right\} \tag{IV-22}$$

$$S_z(r,\omega) = 2\nu \left\{ p' - \frac{\bar{\alpha} E}{(1-\nu)} \sqrt{\frac{\alpha}{2\omega}} \left\{ \frac{\frac{1}{b}\left[k(x_b) + i\ell x(b)\right] + \frac{1}{2}\sqrt{\frac{2\omega}{\alpha}}\, h(x)}{h(x_b)} \right\} \right. \tag{IV-23}$$

where $S_r(r,\omega)$ etc. are the responses to a sinusoidal surface temperature of unit amplitude. It has been assumed in the previous equations that the temperature of the thin casing is uniform and is equal to the ambient temperature.

At the center of the cylinder where $r = 0$ the stress components may be obtained from Equations (IV-5) through (IV-7) by utilizing the limiting values of the integrals discussed previously. Hence,

$$S_r(0,\omega) = -p' + \frac{\bar{\alpha} E}{(1-\nu)\, h\left(\sqrt{\frac{\omega}{\alpha}}\, b\right)} \left\{ -\frac{1}{b}\sqrt{\frac{\alpha}{2\omega}} \left[k\left(\sqrt{\frac{\omega}{\alpha}}\, b\right) + i\ell\left(\sqrt{\frac{\omega}{\alpha}}\, b\right)\right] - \frac{1}{2}(1+i) \right\} \tag{IV-24}$$

$$S_\theta(0,\omega) = -p' - \frac{\bar{\alpha} E}{(1-\nu)\, h\left(\sqrt{\frac{\omega}{\alpha}}\, b\right)} \left\{ \frac{1}{b}\sqrt{\frac{\alpha}{2\omega}} \left[k\left(\sqrt{\frac{\omega}{\alpha}}\, b\right) + i\ell\left(\sqrt{\frac{\omega}{\alpha}}\, b\right)\right] - \left[\frac{1}{2}(1+i) - 1\right] \right\} \tag{IV-25}$$

$$S_z(0,\omega) = 2\nu \left\{ p' - \frac{\bar{\alpha} E}{(1-\nu)\, h\left(\sqrt{\frac{\omega}{\alpha}}\, b\right)} \left[\frac{1}{b}\sqrt{\frac{\alpha}{2\omega}} \left(k\left(\sqrt{\frac{\omega}{\alpha}}\, b\right) + i\ell\left(\sqrt{\frac{\omega}{\alpha}}\, b\right)\right) + \frac{1}{2} \right] \right\} \tag{IV-26}$$

where p' is given by Equation (IV-20).

Equations (IV-20) through (IV-23) are the frequency response functions for the individual stress components. They are presented in Figures 6 and 7 for the material properties listed in Table 1. The time dependent stresses are easily obtained by multiplying the real parts of Equations (IV-21) through (IV-23) with $e^{i\omega t}$. Figure 8 shows the time histories of S_r and S_θ at various positions in the cylinder as

$$S_i(r,\omega) = |S_i(r,\omega)| \cos(\omega t + \phi), \tag{IV-27}$$

while Figure 9 displays the variations of the stress components through the cylinder at fixed times.

V. STRESSES IN THE CASE

It has been assumed in the foregoing that the temperature of the case is uniform through its thickness and is equal to the ambient temperature.

The case stresses are given in Reference [8] for the general plane strain conditions used here as

$$S_{r_c} = \frac{b^2 p'}{(c^2-b^2)}\left(1+\frac{c^2}{r^2}\right) + \frac{\bar{\alpha}_c E_c}{(1-\nu_c)(c^2-b^2)}\left(1-\frac{b^2}{r^2}\right)\int_b^c rU(r)dr - \frac{\bar{\alpha}_c E_c}{(1-\nu_c)r^2}\int_b^r rU(r)dr \quad , \tag{V-1}$$

$$S_{\theta_c} = \frac{b^2 p'}{(c^2-b^2)}\left(1+\frac{c^2}{r^2}\right) + \frac{\bar{\alpha}_c E_c}{(1-\nu_c)(c^2-b^2)}\left(1+\frac{b^2}{r^2}\right)\int_b^c rU(r)dr + \frac{\bar{\alpha}_c E_c}{(1-\nu_c)r^2}\int_b^c rU(r)dr - \frac{\bar{\alpha}_c E_c U(r)}{1-\nu_c} \quad , \tag{V-2}$$

and

$$S_{z_c} = \frac{2\nu b^2 p'}{(c^2-b^2)} + \frac{2\nu_c \bar{\alpha}_c E_c}{(1-\nu_c)(c^2-b^2)}\int_b^c rU(r)dr - \frac{\bar{\alpha}_c \nu_c E_c U(r)}{(1-\nu_c)} \tag{V-3}$$

where c is the outer radius of the case. As a consequence the stress components at the interface between case and cylinder (r=b) become:

$$S_{r_c} = p' \quad , \tag{V-4}$$

$$S_{\theta_c} = p' \left(\frac{b^2+c^2}{c^2-b^2}\right) \quad , \tag{V-5}$$

and

$$S_{z_c} = 2\nu p' \left(\frac{b^2}{(c^2-b^2)}\right) \tag{V-6}$$

with p' given by Equation (IV-4).

Utilizing the material properties presented in Table 1 for a 22 in. (55.9 cm) propellant cylinder in a 0.1 in. (0.25 cm) thick steel case the interlaminar stresses in the case are evaluated as:

$$S_{r_c} = 0.40 \cos \left(\frac{2\pi t}{T} - 0.92\right) \tag{V-7}$$

$$S_{\theta_c} = 43.71 \cos \left(\frac{2\pi t}{T} -0.92\right) \tag{V-8}$$

$$S_{z_c} = 21.22 \cos \left(\frac{2\pi t}{T} -0.92\right) \tag{V-9}$$

where T = 24 hrs is the period of the diurnal cycle.

VI. MEAN AND VARIANCE OF THERMAL STRESSES

The annual thermal cycle consists of a yearly mean temperature on which a sinusoidal variation with a frequency of one per year is superimposed. Thermal stress is produced by the difference between the stress free temperature (the temperature at the time of construction) and its momentary value. If it is assumed that the cylinder is constructed at the yearly mean temperature then only the slowly varying annual cycle will be considered in the determination of the similarly varying mean values of the stress components. While these could be calculated from the relations developed for alternating surface temperatures, Equations (IV-20) through (IV-23), little error is incurred if it is assumed that the temperature throughout the cylinder is constant and varies sinusoidally in time with a frequency of 1/8760 hrs. It can be seen from Equations (II-26) and (II-27) and the series, Equations (II-17) and (II-18), that the temperature at the center of a 22 in.

(55.9 cm) diameter propellant cylinder [α = 1.27 in.^{2}/hr (0.228 x 10^{-6} m^2/s)] is for all practical purposes unity while the time delay is of the order of 30 hrs, a small amount compared to the yearly period of 8760 hrs.

Hence the annual temperature variation throughout the cylinder will be characterized as

$$U_y(r,t) = U_y \sin\left(\frac{2\pi t}{8760}\right) \quad . \qquad \text{(VI-1)}$$

Substituting into Equations (IV-5) through (IV-8), performing the integrations and simplifying, the mean values of the stress components are obtained as follows:

$$S_{rm} = -p_m', \; S_{\theta m} = -p_m' \text{ and } S_{zm} = 2\nu p_m' \qquad \text{(VI-2)}$$

where

$$p_m' \; \frac{E\,U_y\left[\bar{\alpha}\,(1+\nu) - \bar{\alpha}_c\,(1+\nu_c)\right]}{(1+\nu)(1-2\nu) + (1-\nu_c^{\,2})\,\frac{bE}{h_c E_c}} \sin\left(\frac{2\pi t}{8760}\right) \quad . \qquad \text{(VI-3)}$$

Equations (V-2) and (V-3) may also be derived as a limiting case of the sinusoidal thermal input by permitting the frequency, ω, to assume very small values $\left(\omega = \frac{2\pi}{8760}\,\frac{\text{rad}}{\text{hr}} \text{ for the annual cycle}\right)$. Utilizing the first two terms of the series expansions of ber_1, bei_1, ber and bei [Equations (II-7), (II-8), (II-17), and (II-18)] the **integrals** of Equations (IV-15) and (IV-16) reduce to

$$\int_0^r r\,\text{ber}\sqrt{\frac{\omega}{\alpha}}\,r\,dr = \frac{r^2}{2} \qquad \text{(VI-4)}$$

and

$$\int_0^r r\,\text{bei}\sqrt{\frac{\omega}{\alpha}}\,r\,dr = \frac{\omega}{\alpha}\,\frac{r^4}{16} \qquad \text{(VI-5)}$$

and the function h of Equation (IV-17) when higher order terms containing ω are dropped becomes

$$h\left(\sqrt{\frac{\omega}{\alpha}}\, r\right) = 1 + \frac{\omega}{\alpha}\frac{r^2}{4}\, i \quad . \qquad \text{(VI-6)}$$

Substituting these expressions into the equations for the stresses and neglecting the imaginary terms (they alone contain ω) Equations (VI-2) and (VI-3) are again obtained.

The value of p_m' has been calculated for the material properties shown in Table 1 as

$$p_m' = 1.171\, U_y \sin\left(\frac{2\pi t}{8760}\right) \quad . \qquad \text{(VI-7)}$$

If there is a difference, U_m, between the stress free temperature and the yearly mean an additional term analogous to Equation (VI-3), but with U_m replacing $U_y \sin\left(\frac{2\pi t}{8760}\right)$ would be added to Equation (VI-3).

The narrow band random stresses produced by the diurnal cycle are superimposed on these slowly varying mean values. The variances of the stress components may be calculated in a manner analogously to the temperature variance as shown in Section III.

The power spectrum of any one of the stress components is given directly from the power spectrum of surface temperatures. From Equation (III-1)

$$W_{sj}(f) = |S_j(r,f)|^2\, W_{u_i}(f) \qquad j = r, \theta, z \quad . \qquad \text{(VI-8)}$$

While the variance of a stress component may be obtained by numerical integration as

$$E\left[S_j^2(r)\right] = \int_0^\infty |S_j(r,f)|^2\, W_{u_i}(f)\, df \quad . \qquad \text{(VI-9)}$$

For the band limited white noise surface temperature input with central frequency expressed in cycles per hour and with $W_{u_i}(f)$ = constant, the one sided power spectral density in $(°F)^2$/cy/hr or $(°C)^2$/cy/hr is

$$E\, S_j^2(r) = W_{u_i} \int_{f_1}^{f_2} |S_j(r,f)|^2\, df \quad . \qquad \text{(VI-10)}$$

The standard deviations which are the square roots of the variances have been calculated with a rectangular band limited white noise centered over the 24-hour period as shown in Figure 5.

In this narrow range the frequency response functions are relatively constant. As a consequence the standard deviations of the temperature and the stress components for a 1°F (0.555°C) surface temperature standard deviation follow the curves of Figure 3 within plotting accuracy. In the particular geometry and material properties chosen, the curves for the amplitudes of the frequency response functions also indicate the standard deviations of the various functions for a one degree Fahrenheit surface temperature standard deviation.

The average frequency of the response to a narrow band excitation may be obtained from random process theory [6].

$$\bar{f}^2 = \frac{\int_{f_1}^{f_2} (f)^2 \, W_{Sj} \, df}{\int_{f_1}^{f_2} W_{Sj} \, df} \qquad \text{(VI-11)}$$

According to Equation (VI-8) the average frequency of the stress components at any point within the cylinder may then be expressed in terms of the frequency response function and the input power spectral density as

$$\bar{f}^2 = \frac{\int_{f_1}^{f_2} f^2 |S_j(r,f)|^2 \, W_{u_i}(f) \, df}{\int_{f_1}^{f_2} |S_j(r,f)|^2 \, W_{u_i}(f) \, df} \quad . \qquad \text{(VI-12)}$$

When the input power spectral density is a constant

$$\bar{f}^2 = \frac{\int_{f_1}^{f_2} f^2 \, |S_j(r,f)|^2 \, df}{\int_{f_1}^{f^2} |S_j(r,f)|^2 \, df} \quad . \tag{VI-13}$$

Integration of Equation (VI-12) for the narrow frequency range of f_1 = 1/23 cy/hr and f_2 = 1/25 cy/hr shows that the average frequency of the output stress components is very close to $\bar{f}$ = 1/24 because the frequency response functions are nearly constant in this range.

VII. DISTRIBUTION OF TEMPERATURE PEAKS

If the ambient surface temperature in addition to being a narrow band random process has Gaussian probability of exceedance the distribution of amplitude peaks may be calculated. The density function of amplitudes follows a Rayleigh distribution [6] of the form

$$f_A(a) \quad \frac{a}{\sigma_u^2} e^{-\frac{1}{2}\left(\frac{a}{\sigma_u}\right)^2} \tag{VII-1}$$

where a is the random amplitude of the diurnal cycle and σ_u is the standard deviation of hourly temperature measurements.

The probability of encountering an amplitude greater than a given value a is the integral of Equation (VII-1)

$$P[A > a] = \int_a^{\infty} \frac{a}{\sigma_u^2} e^{-\frac{1}{2}\left(\frac{a}{\sigma_u}\right)^2} da = e^{-\frac{1}{2}\left(\frac{a}{\sigma_u}\right)^2} \quad . \tag{VII-2}$$

The mean amplitude of the process [9] is

$$\bar{a} = \sqrt{\frac{\pi}{2}} \, \sigma_u = 1.25 \, \sigma_u \tag{VII-3}$$

while the standard deviation of amplitudes, σ_a, may be given in terms of the standard deviation of hourly temperature readings σ_u as

$$\sigma_a = 0.65\ \sigma_u \quad . \tag{VII-4}$$

Expressions analogous to Equations (VII-1), (VII-2), and (VII-3) may be written for the amplitude distributions of stresses and temperatures at various positions in the cylinder by substituting the appropriate deviations, σ_{s_r}, σ_{s_θ}, σ_{s_z}, and σ_u in these equations.

The values of the required standard deviations for a one degree surface temperature standard deviation may be obtained from Figure 3.

VIII. SAMPLE CALCULATIONS

In order to calculate the stress response of a structure to thermal variations, temperature records are necessary. The information required consists of the average temperature at the location of the structure, the amplitude of the annual cycle, and the standard deviation of hourly temperatures.

Assuming that the solid propellant was cast at 160°F (71.1°C) and is deployed at a location where the annual mean temperature equal 60°F (15.6°C), the amplitude of the annual cycle (U_y) = 25°F (13.9°C) while the standard deviation of hourly temperature measurements (σ_u) = 8°F (4.4°C).

Using Equations (VI-7), (VII-3), and Figure 6, the tangential stress at the interface between propellant and steel case will consist of the following components; the slowly varying mean stress

$$S_\theta = -1.171 \left[25 \sin \frac{2\pi t}{8760} - (160 - 60) \right]$$

and the average amplitude of the diurnal cycle

$$\overline{S}_{\theta_a}(b) = 1.25\ \sigma_u \times \sigma_{s_\theta} = 1.25 \times 8 \times 0.41 = 4.1 \text{ psi } (28.3 \text{ KPa})$$

whose standard deviation is

$$\sigma_{\theta_a}(b) = 0.65\ \sigma_u \times \sigma_{s_\theta} = 2.13 \text{ psi } (14.7 \text{ KPa}) \quad .$$

Consequently at summer temperatures the interfacial tangential stress will, on the average, cycle between 83.7 (577) and 121.2 psi (836 KPa) while during the winter, stresses between 113 (779) and 150.5 psi (1038 KPa) will be observed.

With ±3 standard deviations these stresses will have maximum and minimum values of 77.3 (533) and 156.9 psi (1082 KPa).

At an interior point stresses may be calculated in a similar manner if the appropriate standard deviations are used from Figure 6.

IX. CONCLUSIONS

Thermal stresses produced by a narrow band random surface temperature variation in a solid propellant cylinder encased in a steel shell have been calculated.

A computer program has been developed for the calculation of stress response to sinusoidal and random thermal inputs.

It has been found that the diurnal temperature cycle produces cyclic stresses whose amplitude remains essentially constant throughout the cylinder. This is in contrast to the variations of the temperature which are attenuated at the interior.

It is expected that this behavior is geometry dependent and that motors with smaller diameters would exhibit different characteristics.

For the large diameter motor investigated here the effects of the diurnal cycle are insignificant compared to those produced by the annual cycle.

In a subsequent study stress-strength interference and probability of failure of such motors may be examined.

TABLE 1. MATERIAL PROPERTIES AND GEOMETRY [10]

	Propellant	Case
Modulus of Elasticity, E, psi (MPa)	500 (3.45)	3×10^7 (20.7×10^4)
Poisson Ratio, ν	0.490	0.250
Coefficient of Thermal Expansion, $\bar{\alpha}$, in./in.-°F (cm/cm-°C)	5.5×10^{-5} (9.9×10^{-5})	6.5×10^{-6} (11.7×10^{-6})
Outer Diameter, b, in. (cm)	22 (55.9)	22.2 (56.4)
Thermal diffusivity, α, in.^{2}/hr (m^2/S)	1.27 (0.228×10^{-6})	

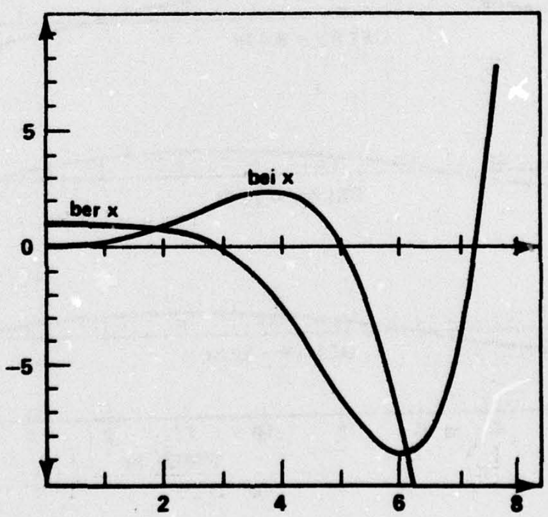

Figure 1. Kelvin functions of order zero: ber(x), bei(x).

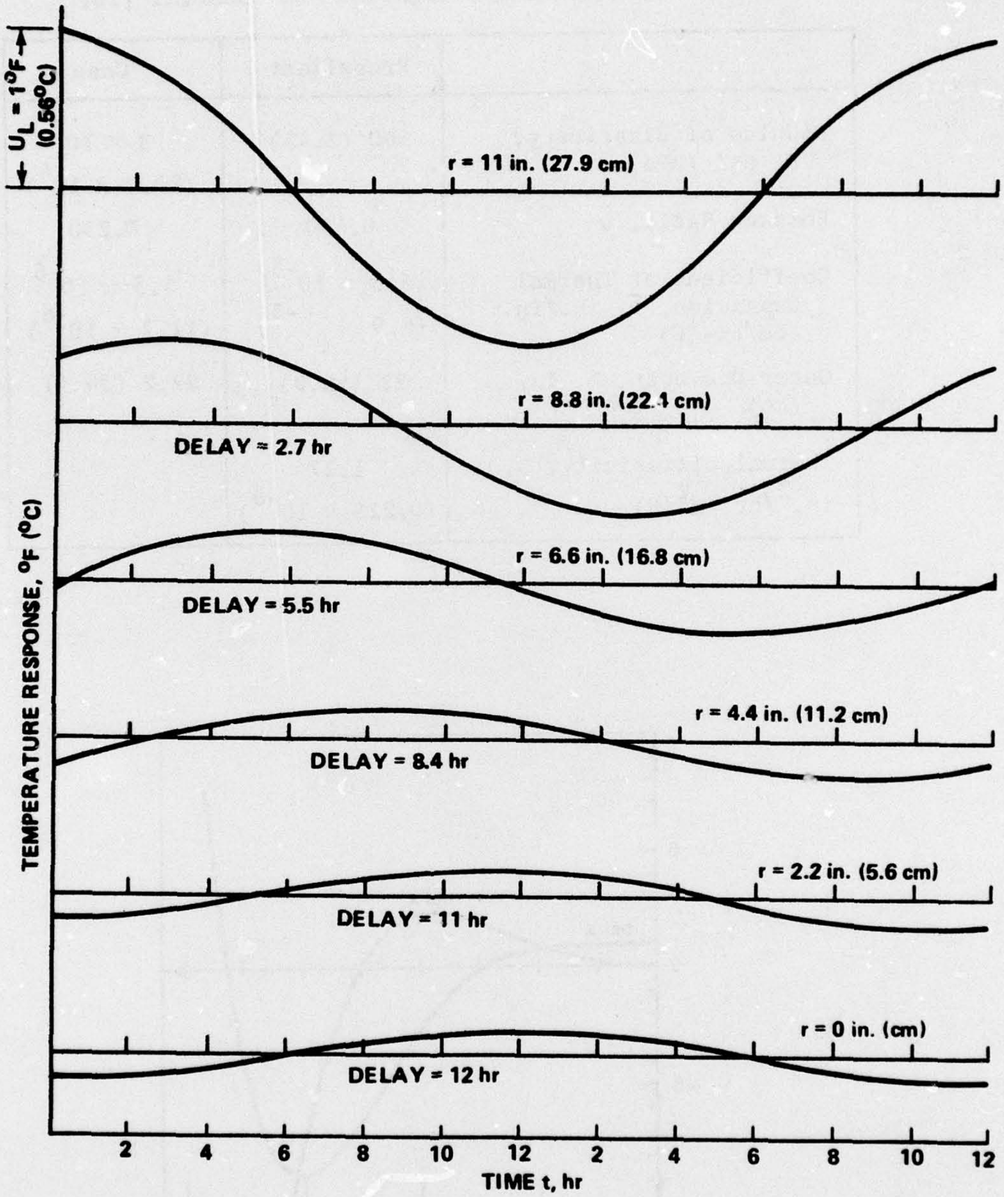

Figure 2. Attenuation and delay of temperature cycle.

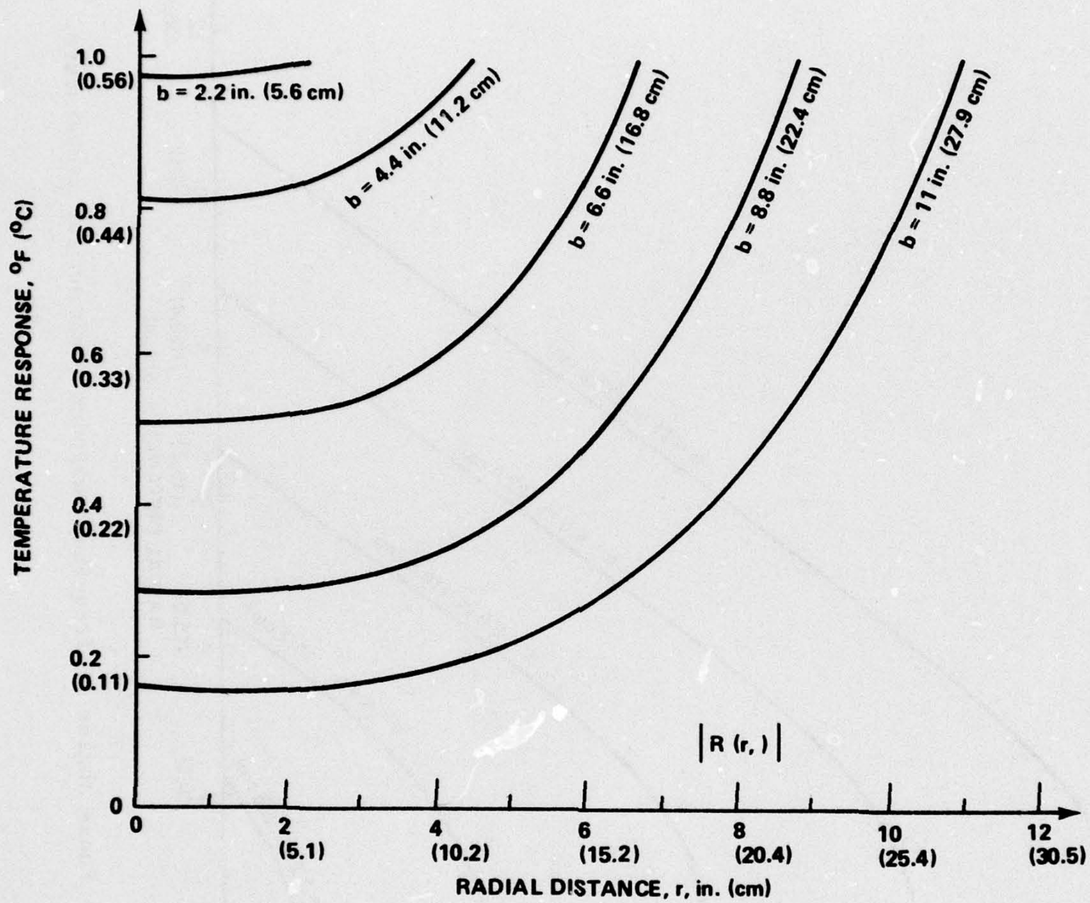

Figure 3. Amplitude of frequency response function for temperature.

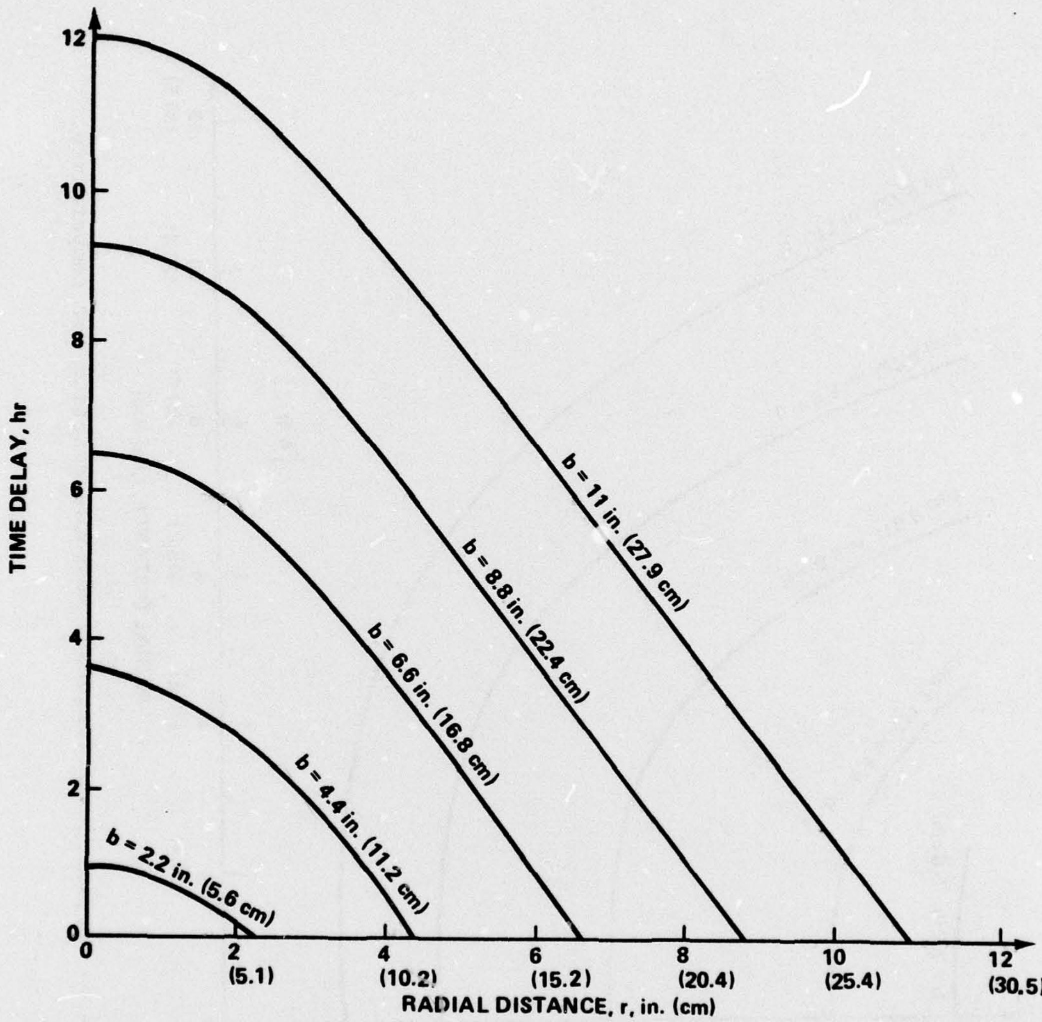

Figure 4. Phase angles of frequency response function for temperature.

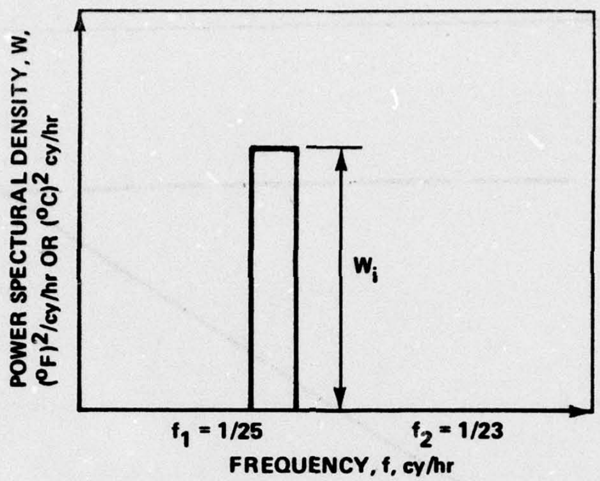

Figure 5. Band limited white noise power spectrum.

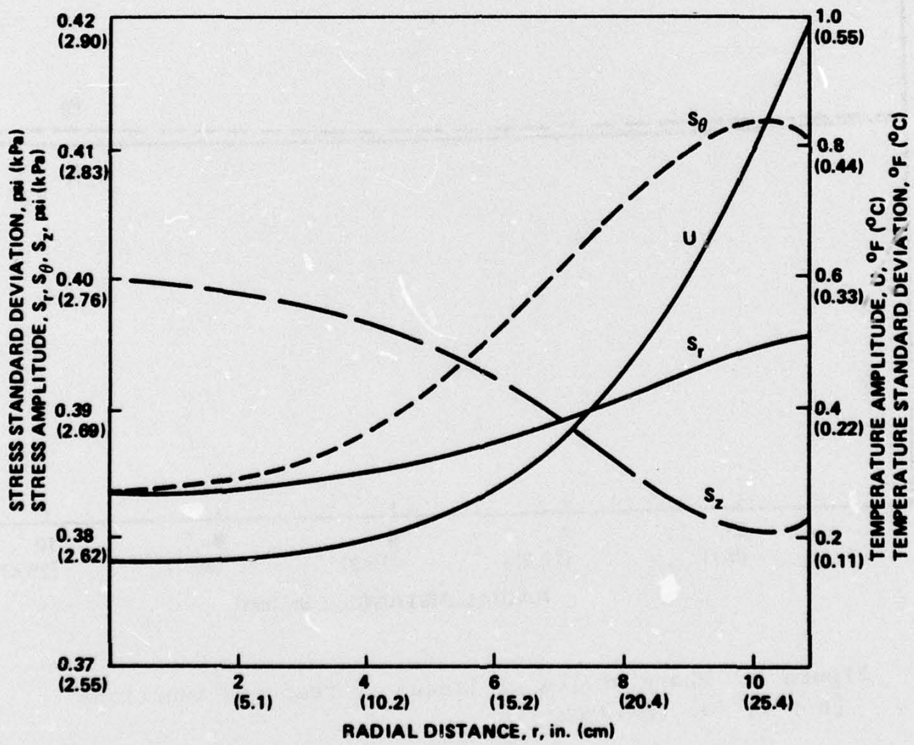

Figure 6. Amplitudes of frequency response functions, standard deviations of response functions for unit surface temperature [b = 11 in. (27.9 cm)].

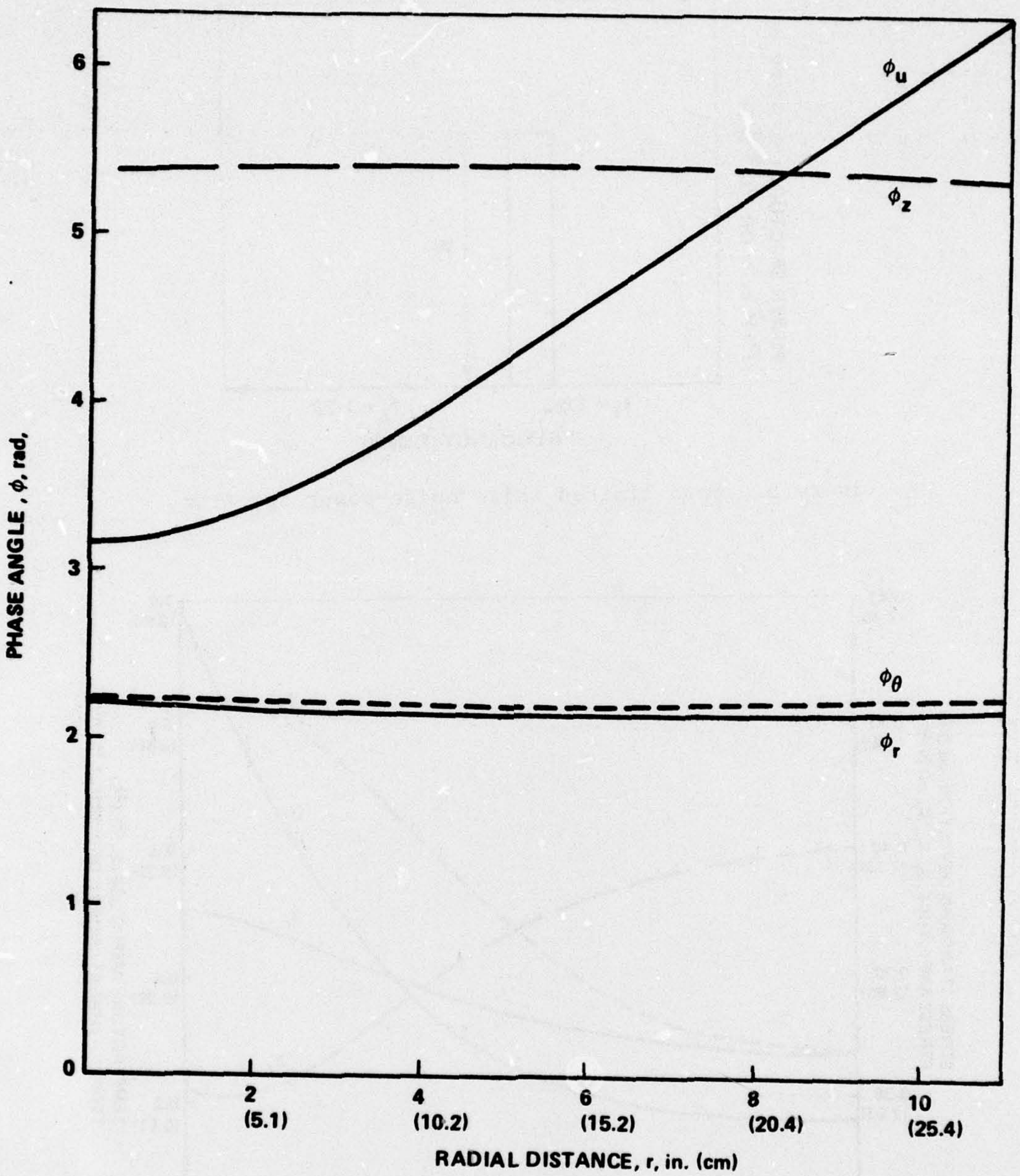

Figure 7. Phase angles of frequency response functions [b = 11 in. (27.9 cm)].

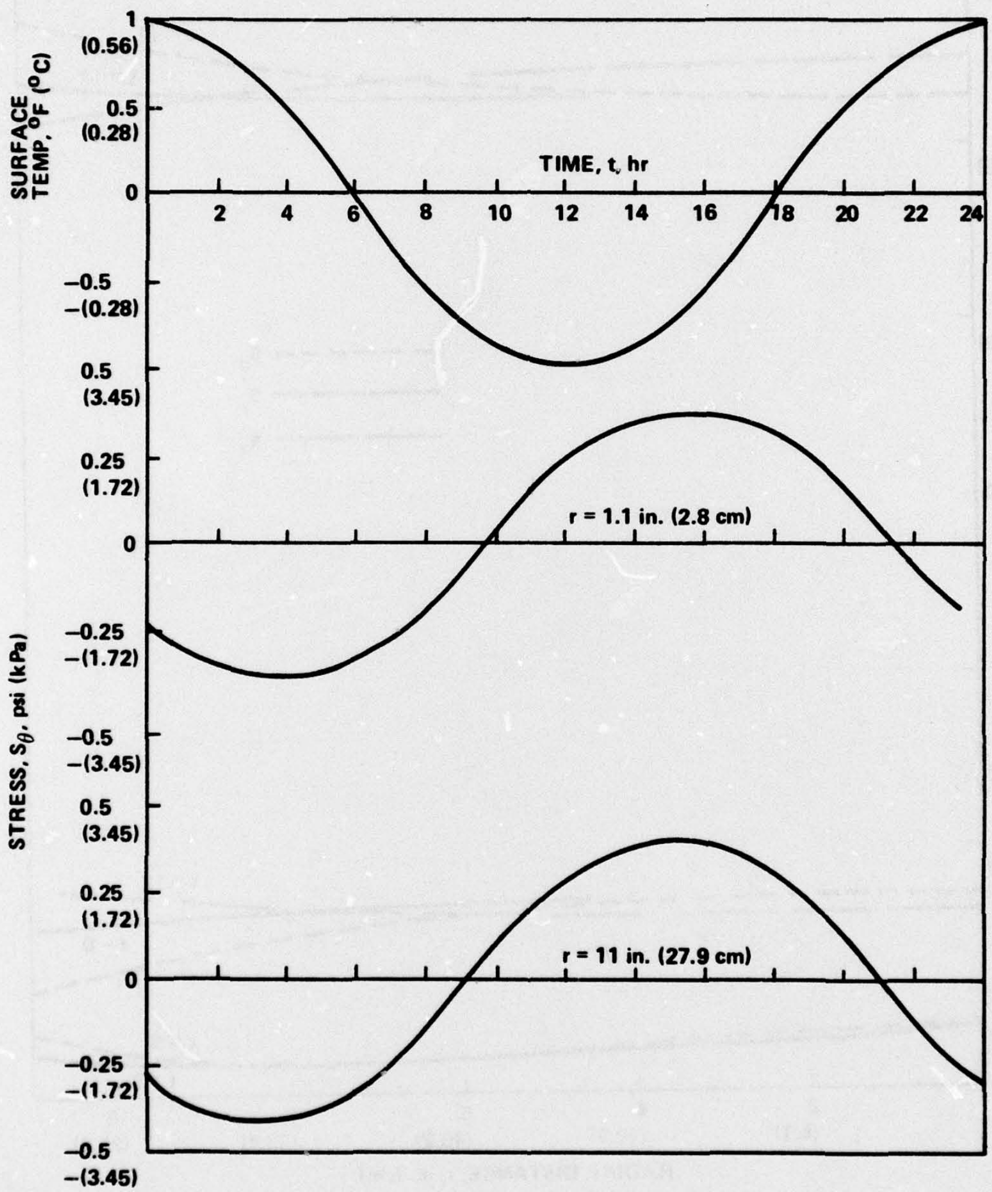

Figure 8. Time history of stress component S_θ.

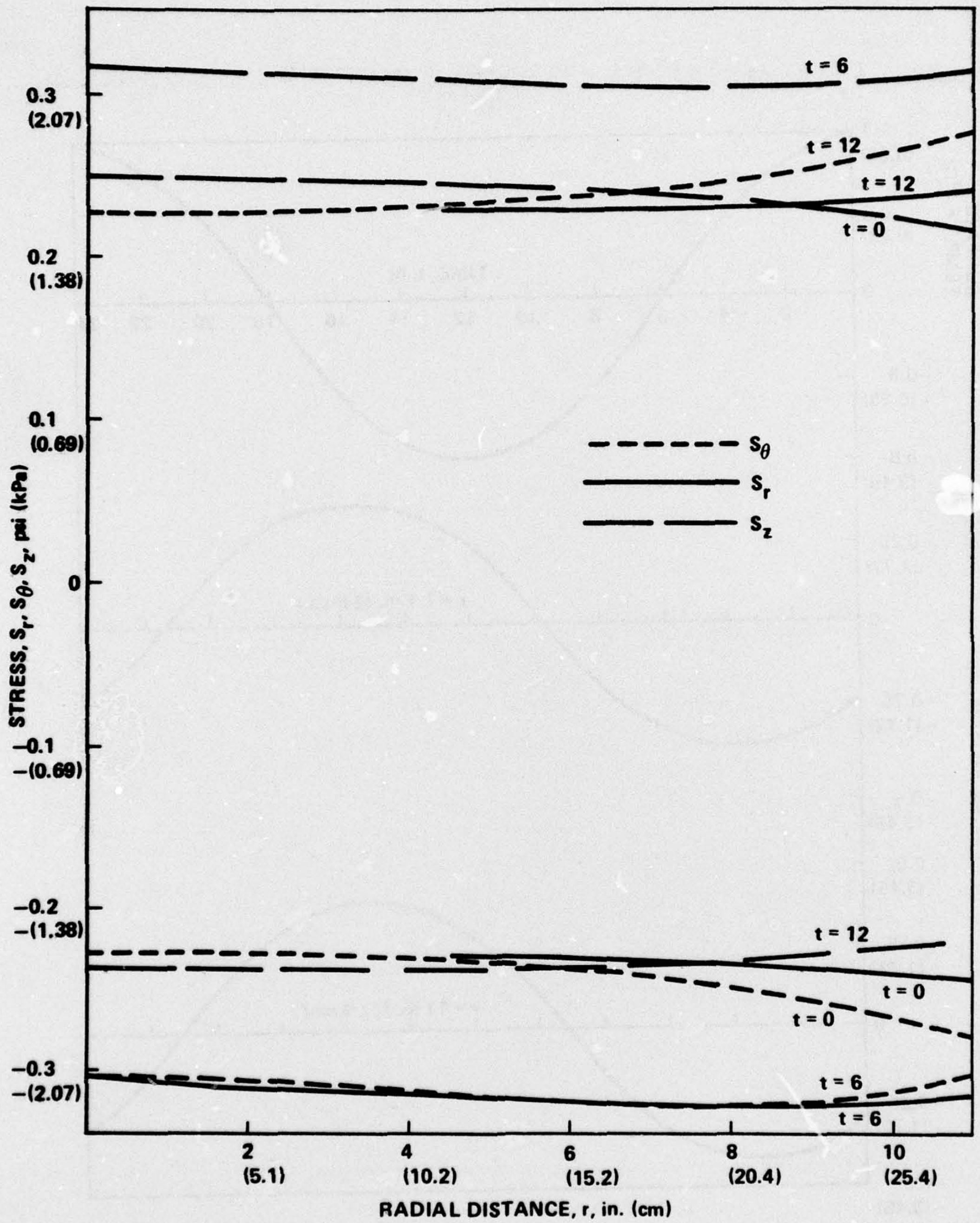

Figure 9. Stress components for fixed times.

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